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ALTITUDE OF EQUILIBRIUM OF AN AIRSHIP.

By Umberto Nobile.

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ALTITUDE OF EQUILIBRIUM OF AN AIRSHIP.*

By Umberto Nobile.

1. In a previous article ("On the Maximum Flight Value of Aeroplanes" in 1917) I established the following formula for determining the maximum ceiling of an airplane:

$$H = \frac{T_o}{0.0055} \left(1 - \rho^{0.128}\right) \qquad (1)$$

in which ρ represents the ratio between the minimum power required for flying at ground level, and the maximum available power (measured at ground level) of an airplane, while the number 0.0055 represents the atmospheric thermal gradient expressed in degrees centigrade per meter and T_o indicates the absolute temperature measured at ground level. The exponent of ρ in formula (1) is the value of the expression $\frac{2}{3} \frac{G\ R}{1 - GR}$ in which G is the value of the atmospheric thermal gradient and R the characteristic constant of the air.

It is needless to tell here how formula (1) was obtained. It is only necessary to remember that it is based on the following expression found for the ratio ϵ between the density of the air at a given altitude and the density at ground level:

$$\epsilon = \left(1 - \frac{G}{H}\right)^{\frac{1}{GR} - 1} \qquad (2)$$

* From "Rendiconti Tecnici," November 15, 1924, pp. 6-15.

2. We now propose to apply to an airship a procedure analogous to that followed in the case of the airplane, to the end of establishing a general formula enabling the calculation of the maximum altitude attainable statically. Assuming the engines to be stopped, the airship at said maximum altitude must be in equilibrium with the surrounding air; in other words, the lifting force must exactly equal the total weight.

This total weight P_H, however, constituted (own weight, rigging, crew, fuel, ballast and useful load) may invariably be expressed in terms of the maximum displacement,

$$P_H = \Psi_H V,$$

in which, assuming V to be expressed in cubic meters and P_H in kilograms, Ψ_H will be expressed in kilograms per cubic meter and may be defined as the <u>mean specific weight of the airship</u>. (This term is new, but is adopted for the sake of brevity and clarity of expression.)

As far as the total lifting force F_H which must equilibrate P_H is concerned, it is found by simply multiplying the volume V occupied by the gas by the difference between the specific weights of the air and of the gas in question:

$$F_H = (d_a - d_g)_H V = f_H V$$

In the condition of equilibrium mentioned above, we assume, for simplicity's sake, that there is no appreciable difference between the temperatures and the pressures of the air and gas.

Consequently, by calling f_H and f_o, respectively, the values of the specific lifting force at altitude H and O, we obtain, bearing in mind the signification of the symbol ϵ,

$$F = \epsilon f_o\ V = f_o\ V \left(1 - \frac{G}{T_o}\ H\right)^{\frac{1}{GR} - 1} .$$

Since P_H must be equal to F_H at the altitude of equilibrium, we get

$$\psi_H\ V = f_o\ V \left(1 - \frac{G}{T_o}\ H\right)^{\frac{1}{GR} - 1}$$

from which is obtained the value of H:

$$H = \frac{T_o}{G} \left(1 - \frac{\psi_H}{f_o}\right)^{\frac{1-GR}{GR}} \tag{3}$$

which is thus determined as a function of the atmospheric gradient, of the specific weight of the airship, of the absolute temperature at ground level and of the specific lifting force measured also at ground level. The value of the specific lifting force depends not only on the temperature, on the pressure and on the humidity of the air and gas, but also on the degree of purity of the gas itself.

This purity may implicitly be defined by indicating the specific lifting force of the gas φ in dry air, at $0^\circ C$ and 760 mm barometric pressure.

In this case, apart from the degree of humidity, the specific lifting force f_o for a temperature T_o and a pressure p_o will be expressed by

$$f_o = \frac{273}{760}\ \frac{p_o}{T_o}\varphi = 0.359\ \frac{p_o}{T_o}\varphi$$

By substituting this value in formula (3) we finally obtain the general expression:

$$H = \frac{T_o}{G}\left[1 - \frac{\psi_H \; T_o}{0.359 \; p_o \; \varphi}\right]^{\frac{GR}{1-GR}} \qquad (4)$$

3. Formula (4) gives the following laws of variation of the altitude of equilibrium H:

a) The altitude of equilibrium varies inversely as the thermal gradient and vice versa (as for airplanes);

b) The altitude of equilibrium increases with the atmospheric pressure, measured at the ground, with the degree of purity of the gas;

c) The altitude of equilibrium increases, as the specific weight of the airship decreases;

d) The altitude of equilibrium increases, as the temperature at ground level decreases, as clearly shown by the numerical applications.

Consequently, if, with a given airship, it is desired to attain statically the highest possible altitude, it is expedient not only to reduce the total weight to a minimum, but also to start the flight with very pure gas, high barometric pressure and low temperature.

In order to demonstrate more clearly the importance of the various elements in the determination of the altitude H, Figs. 1, 2 and 3 were traced, for a gas with a specific lifting force of 1.142 kg per cubic meter, at 0^o and 760 mm and in dry air; in other words, a specific weight of 150 grams per cubic meter.

In Fig. 1 p_o was assumed to be 760 mm. After making the values of ψ on the abscissa, the curves of variation of the values of H were traced for various values of T_o.

In Fig. 2 the variations of H, in terms of the temperature were shown by marking on the abscissa the values of said temperature. The curves of the H values are based on the different values of ψ_H and of p_o.

Finally, in Fig. 3, the values of the pressure p_o were marked on the abscissa. The curves of the H values were also traced in this case by considering a certain number of values of the other two elements ψ and T_o.

It may be deduced from the figures that, as the altitude increases, the influence of the temperature decreases. This, however, is not the case with the pressure.

As regards the influence of the ψ variations, the trend of the curves demonstrates that, as their value decreases, the gain in altitude increases faster in proportion.

We will consider, for example, an airship having (as may actually be the case for a high altitude type) a weight of 0.5 kg per cubic meter and, assuming that the aggregate weight of the rigging, crew, useful load, reserve ballast and fuel necessary for completing the flight, totals a further 0.2 kg per cubic meter, we get an aggregate specific weight $\psi = 0.7$.

From Fig. 1 we deduce, therefore, for a ground level temperature of 15°, that $H = 4450$ m.

If we now propose to lighten the structure (assuming this to be possible) to the end of obtaining a gain in altitude of 1500 meters, we deduce from the same figure that the new ψ must have the value of about 0.57, that is, the specific weight of the airship must be reduced from 0.5 to 0.37 per cubic meter, that is, a reduction of 26%.

This high value clearly demonstrates the great difficulties opposing any increase in the maximum flight altitude of a given type of airship.

Below is a table of the values on which the figures were based and which enables the immediate reading of the maximum variations which may be encountered in the maximum ceiling obtainable statically.

Thus, for instance, in the case of the airship considered above, ψ being 0.7, we find that the maximum altitude may vary from a minimum of 4092 meters, corresponding to 15° and 750 mm, to a maximum of 4784 meters, corresponding to -15°C and 770 mm.

Values of H.

	p	-15°	0	+15°
ψ = 0.6	750	5807	5669	5501
	760	5912	5781	5620
	770	6015	5882	5738
ψ = 0.7	750	4570	4347	4092
	760	4678	4461	4214
	770	4784	4571	4335

Values of H (Cont.)

	p	-15°	0	$+15^{\circ}$
ψ = 0.8	750	3469	3169	2837
	760	3580	3286	2961
	770	3688	3393	3086
ψ = 0.9	750	2474	2104	1701
	760	2587	2224	1829
	770	2699	2334	1957

4. The formula, established above for the determination of the static ceiling of an airship, presupposes a perfect thermal equilibrium between the gas in the hull and the surrounding atmosphere.

However, even when such thermal equilibrium exists at ground level at the time of ascent, it generally comes about that no sooner has the ascent begun than the equilibrium is disturbed, both on account of the variation of the air temperature and because of the absorption of heat corresponding to the work done by the gas in expanding. It is easy to understand that, in general, the gas tends to cool more rapidly than the surrounding air. In fact, let us assume that the ascent from zero to H altitude is <u>instantaneous</u>, or, if preferred, that the outer envelope is <u>absolutely impermeable</u> to heat, so that, during the ascent, no exchange of heat occurs between the air and the gas.

In passing from zero altitude, where the pressure is p_o, to H altitude where the pressure is p, the gas undergoes an adiabatic expansion which makes its absolute temperature decrease from T_o to T. As is well known, the following relation then exists between the temperatures and pressures:

$$\frac{T_o}{T} = \left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}}$$

in which γ, as usual, represents the ratio between the specific heats of the gas at constant pressure and at constant volume.

For pure hydrogen $\gamma = 1.412$ and, consequently,

$$\frac{T_o}{T} \left(\frac{p_o}{p}\right)^{0.291}$$

On applying this formula to the mean values of the pressure at different altitudes, we find:

$T = T_o$	for	$H = 0$	meters
" $= 0.964\ T_o$	"	" $= 1000$	"
" $= 0.929$ "	"	" $= 2000$	"
" $= 0.896$ "	"	" $= 3000$	"
" $= 0.864$ "	"	" $= 4000$	"
" $= 0.833$ "	"	" $= 5000$	"
" $= 0.803$ "	"	" $= 6000$	"

from which it follows that in the hypothesis of adiabatic expansion, the gas should cool 0.032 T_o degrees centigrade for every 1000 meters of ascent, that is:

8.8° for $T_o = 273^\circ$

9.3° " " $= 273^\circ + 15^\circ$

9.7° " " $= 273^\circ + 30^\circ$

Since for every 1000 meters ascent, the surrounding air cools 5.5°

on an average, it follows from the above hypothesis that the difference between the air temperature and gas temperature would increase with the altitude precisely 4°, on an average, for every 1000 meters ascent. Correspondingly, assuming that the mass of gas remains constant during the ascent, an increase of weight of $\frac{4}{1000} H \frac{d_a}{d_g} \frac{Q}{T}$ approximately, would occur in the airship, in which formula d_a denotes the density and T the absolute temperature of the air at the altitude H attained by the airship, and d_g the density of the gas at the same altitude.

In actual practice the case considered above is impossible of realization, because no ascent can be made at infinite velocity nor is the envelope impermeable to heat. Consequently, in all cases the cooling of the gas in relation to the surrounding air will be less than the extreme value calculated above.

By letting Δ denote the difference between the temperature of the surrounding air and that of the gas (a difference which, in view of the above considerations, may be extremely small, but always a positive finite quantity), we may state, in general, that the quantity of the heat absorbed by the work of expansion of the gas is equal to the sum of the quantity of heat entering the gas through the envelope, as the result of the thermal difference, and of the quantity of heat freed during the cooling of the gas itself.

If R denotes the characteristic constant of the gas under consideration, and v and T denote respectively, the values of its specific volume and its temperature at the altitude H, the infinitesimal variations of these elements, in correspondence with the variation dH of the altitude, will then be represented by dv and dT. Consequently, E being the mechanical heat equivalent, the quantity of heat absorbed by 1 kilogram of gas, during the expansion dv, will be expressed by

$$\frac{R}{E} T \frac{dv}{v}$$

while the quantity of heat freed during the cooling of the same quantity of gas will be represented by

$$C_v \, dT.$$

If we now indicate by A the coefficient of thermal conductivity of the envelope (that is, the quantity of heat transmitted in one hour through one square meter of its surface, when $\Delta = 1^\circ$ centigrade) by S, the total surface of the envelope in square meters, and by Q, the total weight of the gas in kilograms, we find that the quantity of heat entering each kilogram of gas in the time unit dt, through the envelope, is represented by

$$\frac{A \; S \; \Delta}{Q} \, dt,$$

in which $dt = \frac{dH}{w}$, w indicating the velocity of ascent of the airship.

Consequently, the general equation reads as follows:

$$\frac{R}{E} T \frac{dv}{v} = C_v \, dT + \frac{A \; S \; \Delta}{Q} \; \frac{dH}{w}$$

In considering the particular case of Δ constant, the variation of the gas temperature will be equal to that of the air and, consequently,

$$dT = G\,dH.$$

On assuming, furthermore, that the difference between the gas and air pressures is negligible, the specific volume v of the gas may be expressed, according to formula (2) by

$$v = \frac{1}{d_g} \frac{1}{\left(1 - \frac{G}{T_o} H\right)^{\alpha}},$$

in which d_g represents the gas density with respect to the air and α the known exponent, whose mean value may be assumed to be 5.2. Consequently, we immediately obtain

$$dv = \frac{1}{d_g} \alpha \frac{G}{T_o} \frac{dH}{\left(1 - \frac{G}{T_o} H\right)^{\alpha+1}}$$

$$\frac{dv}{v} = \alpha \frac{G}{T_o} \frac{dH}{1 - \frac{G}{T_o} H}$$

By substituting in equation (5) the above values for dT and $\frac{dv}{v}$, dH is eliminated and we finally obtain

$$\frac{R}{E} \alpha\, G = C_v G + \frac{A\, S\, \Delta}{Q w} \tag{6}$$

and, therefore,

$$w = \frac{AS}{Q\left(\frac{R}{E} \alpha\, G - C_v G\right)} \Delta \tag{7}$$

During ascent with uniform motion, assuming the gas mass to remain constant, the surface S confining the gas will increase with the altitude and, consequently, Δ will decrease.

Analogously, if the ascent is made with the envelope full of gas, S will remain constant while Q decreases. The ratio S : Q, as in the preceding case, will increase as the airship ascends and consequently (also in this case) Δ will diminish as the altitude increases.

Assuming, for simplicity's sake, that the ratio S : Q remains constant and equal to an average value, we may draw the following important conclusions:

a) The thermal difference between air and gas is proportional, for a given airship, to the velocity of ascent;

b) The velocities with which two similar airships must ascend from zero to H altitude, so that the thermal difference will remain Δ degrees, are proportional to the linear dimensions of the airships;

c) In the case of similar airships having the same velocity of ascent, the thermal differences are inversely proportional to the linear dimensions of the airships.

If, at the beginning of the ascent, the temperatures of the air and gas are equal (still assuming w and S : Q to be constant), there will be a period of transition, in view of the fact that the thermal difference must pass from 0 to Δ degrees, but this period is, for obvious reasons, relatively very short.

5. Evaluation of the Coefficient of Thermal Conductivity.- Thus far, there has not been a sufficient number of experiments to enable us to determine, with sufficient approximation, the value and the law of variation of the coefficient of conductivity A for the various types of rubberized fabrics employed in the construction of gas bags. (However, the writer intends to carry out soon a series of experiments along these lines.) Nevertheless, we may establish the order of magnitude of said coefficient by getting it down under the general form:

$$A = \frac{1}{\frac{1}{c_1 + i_1} + \frac{s'}{k'} + \frac{s''}{k''} + \frac{1}{c_2 + i_2}}$$

in which

s' and s'' represent the aggregate thickness of the various layers of cotton and rubber composing the fabric;

k' and k'', the respective coefficients of thermal conductivity;

c_1 and i_1, the coefficients of transmission through conduction and irradiation of the inner lining of the rubberized fabric, generally of raw cotton;

c_2 and i_2, the analogous coefficients for the outer surface, which is generally rubberized and coated with aluminum paint.

We can, for example, assume:

s' = m 0.003

s'' = " 0.0002

k' = " 0.05

$$k'' = m\ 1$$

$$i_1 = \text{"}\ 4$$

$$c_1 = \text{"}\ 3$$

$$i_2 = \text{"}\ 1$$

$$c_2 = \text{"}\ 30^*$$

in which case we get:

$$\frac{1}{c_1 + i_1} = 0.143$$

$$\frac{1}{c_2 + i_2} = 0.033$$

$$\frac{s'}{k'} = 0.006$$

$$\frac{s''}{k''} = 0.0002$$

Consequently, A = 5.5 calories per square meter and per hour.

If the thickness of the envelope is doubled, we find A = 5.32 but if, on the contrary, it is reduced 50%, A = 5.59, which means that the thickness of the envelope (within the practical limits between which it may vary) only very slightly affects the value of the coefficient of conductivity.

In the case of very high velocities, the second term may become negligible and in this case (the last two terms also being assumed to be negligible) the coefficient of conductivity attains the maximum value of approximately 7. On the contrary, if the airship is stationary in the air, we may assume

$$\frac{1}{c_2 + i_2} = 0.20$$

and consequently, A = 2.8.

* This coefficient is the most uncertain. It increases with the velocity of the air flowing along the envelope.

We conclude that, until experimental data are forthcoming, A may roughly be assumed to be 3 for stationary airships and 5.5 for airships in motion.

6. Numerical Example.- In order to calculate the value of the expression

$$\frac{R}{E}\,\alpha\,G - C_v G$$

which is encountered in the denominator of formula (7), let us assume that a mixture of hydrogen and air is contained in the gas bag of the airship, consisting of 85.5 parts of hydrogen and 64.6 parts of air, and weighing 150 grams per cubic meter at 0° and 760 mm. In this case we obtain:

$$C_v = \frac{85.5 \times 2.412 + 64.6 \times 0.169}{150} = 145$$

$$R = \frac{85.5 \times 421 + 64.6 \times 29.27}{150} = 252$$

and, since

$$\alpha = 5.2$$

$$G = 0.0055$$

$$E = 427$$

we obtain

$$\frac{R}{E}\,\alpha\,G - C_v G = 0.0089$$

and, consequently,

$$w = 112\,\frac{A\,S}{Q}\,\Delta$$

If we now assume $A = 5.5$ and $S : Q$ equals 2.5 m² per kilogram, in the case of $V = 18,500\ m^3$, we obtain

$$w = \Delta\ 1540 \text{ meters per hour.}$$

Therefore, in ascending at a velocity of 1540 meters per hour, there will be a difference of 1 degree, while $\Delta = 3.9^\circ$ will correspond to a velocity of 6000 meters per hour.

After the velocity of ascent w has been established and it has been assumed that the airship is statically and thermally balanced at ground level, the formula

$$w = 1540\ \Delta$$

enables the calculation of Δ and, consequently, the vertical thrust or lift (static and dynamic) required for attaining said velocity.

On indicating, as customary, by T and d_a the temperature and density of surrounding air and by d_g the corresponding temperature of the gas, the value of said lift is given by the formula

$$Q \frac{\Delta}{T} \frac{d_a}{d_g} = \sim 8.6\ Q \frac{\Delta}{T}$$

from which it follows that Q, being constant, the lift must be inversely proportional to the absolute temperature and, consequently, that, in order to maintain a uniform ascending speed, the lift must gradually increase as the airship ascends.

At the altitude of fullness (when the expanding gas just fills the envelope) $Q : d_g$ equals V, the maximum displacement of the airship, and in this case, the lift is measured by

$$V \frac{\Delta}{T} d_a .$$

From the above considerations, it follows that, in the case of an airship, in perfect static and thermic equilibrium at ground level, ascending dynamically, it attains the altitude of fullness higher than that calculated with formula (4) for the static ceiling. Said altitude will increase with the ascending speed or, in other words, with the dynamic lift.

If, after attaining the altitude of fullness, the airship continues to ascend, there will be a loss of gas and a resulting loss of lifting force. In this case, if the dynamic lift cannot be increased by the same amount, a braking force is created, causing the velocity of ascent to decrease rapidly until it becomes zero. If, on the contrary, after attaining the altitude of fullness, the ascending motion is arrested, the thermal difference, which was constant up to that moment, will gradually decrease. On account of the heating of the gas, the lifting force increases and consequently, if it is desired to maintain the same altitude, it is necessary to decrease the dynamic lift. However, after the thermal equilibrium between gas and air has been reached, the value of the lift is still positive, because the airship is above the altitude of static and thermal equilibrium. This signifies, after all, a static increase of weight resulting from a corresponding loss of gas.

The gas loss and the consequent increase in weight may be prevented, however, by making the airship descend rapidly, after the altitude of fullness is reached. This point may, in practice, assume considerable importance and is therefore worth noting.

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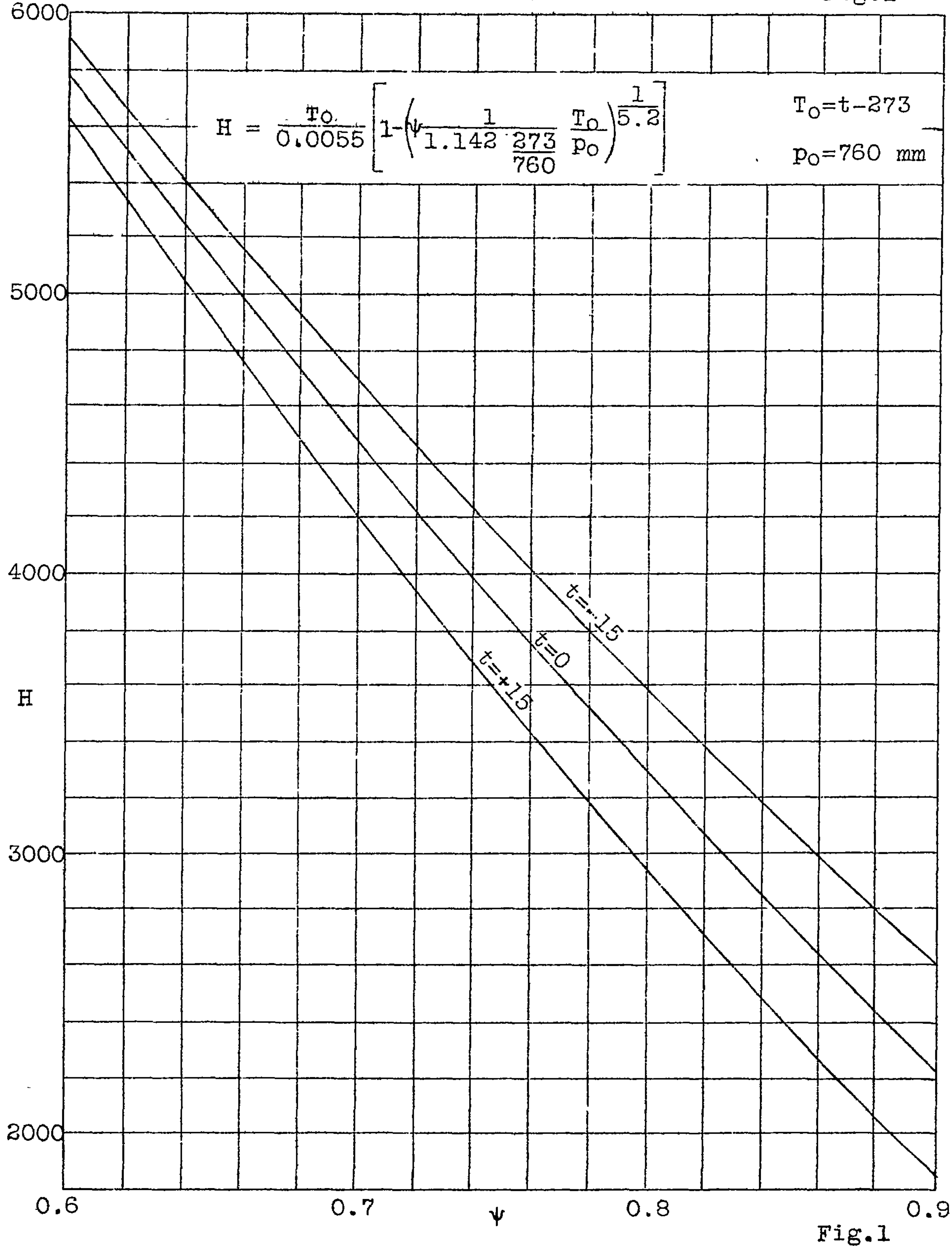
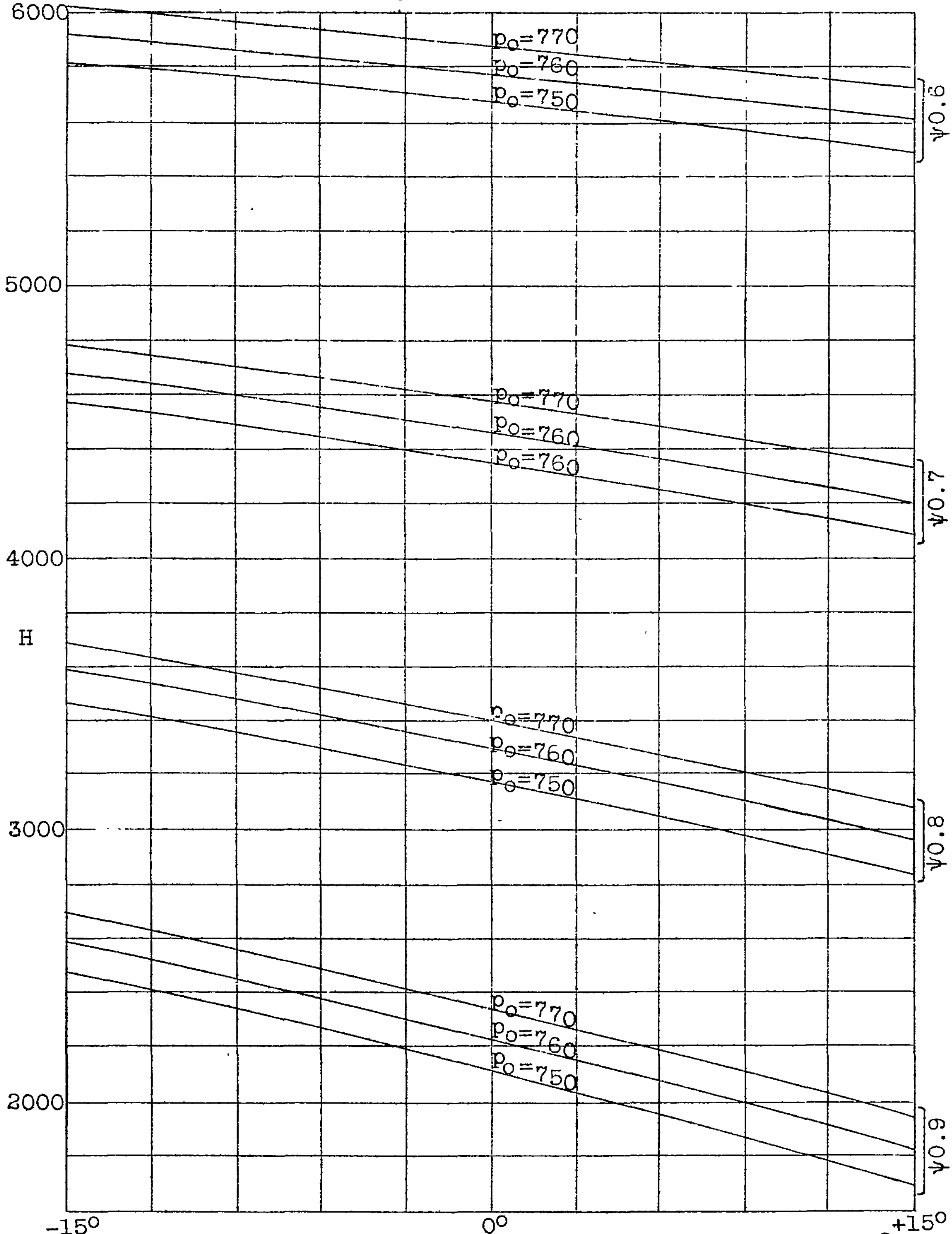

$H = \frac{T_0}{0.0055}\left[1-\left(\psi\frac{1}{1.142\,\frac{273}{760}}\,\frac{T_0}{p_0}\right)^{\frac{1}{5.2}}\right]$
$T_0=t-273$
$p_0=760$ mm
t=-15
t=0
t=+15
6000
5000
4000
3000
2000
H
0.6
0.7
0.8
0.9
ψ

Fig.1


6000
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H
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t
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Fig.2

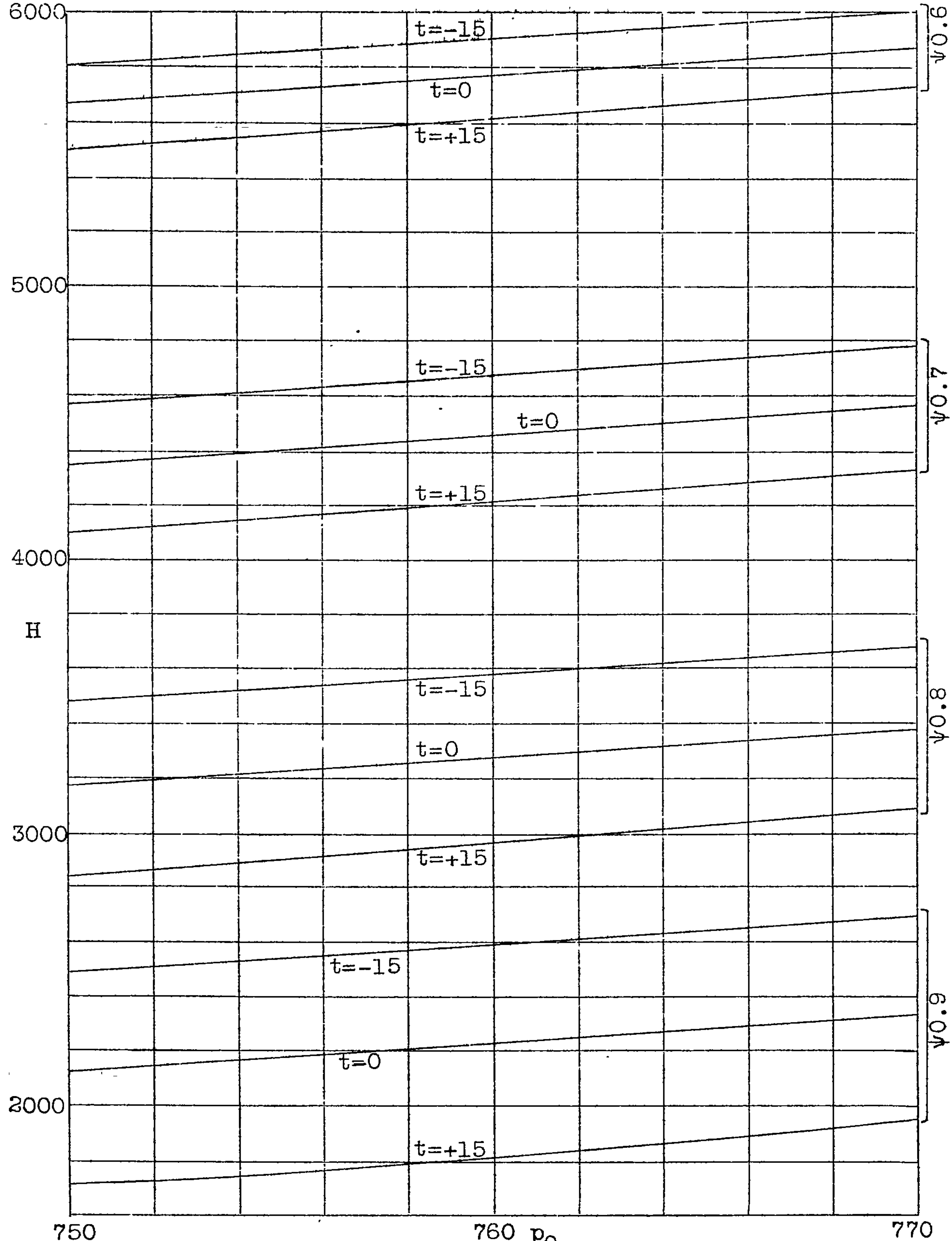

Fig.3